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Generating Unimodular Matrix in Python for Solving Systems of Linear Equations

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ABSTRACT *– Linear equation systems with integer solutions are widely used in modern computing fields such as cryptography and optimization, but conventional methods often produce inaccurate decimal solutions. To address this issue, this research developed a Python program based on NumPy that can efficiently generate unimodular matrices. The method involves three main stages: initializing an upper triangular matrix with diagonal elements of ±1, filling non-diagonal elements with random integers, and transforming the matrix through elementary row operations. Test results show that the program successfully generates unimodular matrices of sizes 4×4 to 9×9 with perfect accuracy (determinant exactly ±1), an average computation time of 0.5 seconds for a 4×4 matrix, and efficient memory usage (under 20 MB). The solutions to the linear equations are always exact integers, meeting the requirements for high-precision computation. This implementation not only provides a practical solution for integer linear equation systems but also opens up opportunities for applications in cryptographic algorithm development and optimization techniques that require absolute precision. The findings of this research confirm that numerical computation approaches can produce both accurate and efficient mathematical solutions.*

Keywords *- Unimodular Matrix, Systems of Linear Equations, Python, NumPy, Determinants*

Generasi Matriks Unimodular dengan Python untuk Solusi Sistem Persamaan Linear

ABSTRAK *–Sistem persamaan linear dengan solusi bilangan bulat banyak dibutuhkan dalam bidang komputasi modern seperti kriptografi dan optimasi, namun metode konvensional sering kali menghasilkan solusi desimal yang tidak tepat. Untuk mengatasi masalah ini, penelitian ini mengembangkan program Python berbasis NumPy yang mampu menghasilkan matriks unimodular secara efisien. Metode yang digunakan melibatkan tiga tahap utama: inisialisasi matriks segitiga atas dengan elemen diagonal ±1, pengisian elemen non-diagonal dengan bilangan bulat acak, serta transformasi melalui operasi baris elementer. Hasil pengujian menunjukkan bahwa program ini berhasil menghasilkan matriks unimodular berukuran 4×4 hingga 9×9 dengan akurasi sempurna (determinan tepat ±1), waktu komputasi rata-rata 0.5 detik untuk matriks 4×4, dan penggunaan memori yang efisien. Solusi persamaan linear yang dihasilkan selalu berupa bilangan bulat tepat, memenuhi kebutuhan komputasi presisi tinggi. Implementasi ini tidak hanya menyediakan solusi praktis untuk masalah sistem persamaan linear integer, tetapi juga membuka peluang aplikasi dalam pengembangan algoritma kriptografi dan teknik optimasi yang memerlukan presisi mutlak. Temuan penelitian ini menegaskan bahwa pendekatan komputasi numerik dapat menghasilkan solusi matematis yang akurat sekaligus efisien.*

Kata Kunci *– Matriks unimodular, Sistem persamaan linear, Python, NumPy, Determinan.*

1. INTRODUCTION

Linear algebra studies Systems of Linear Equations (SLE) and matrix. SLE with variables $x_{1,\ldots,\ldots,\ldots,\ldots}X2$ stated as $a_1x_1 + \ldots + a_nx_n = b$, where $\in \mathbb{R}$ There are two methods for finding solutions to systems of linear equations: the matrix inverse method and Cramer's rule. However, both methods only apply to non-singular square matrix (determinant ≠ 0). Unimodular matrix (those with a determinant of ±1) ensure that the solution to the system of linear equations consists of integers. This research project involves generating unimodular matrix using Python.

There are several methods for solving a system of linear equations (SLE), including the matrix inverse method and Cramer's rule. However, these methods only apply to non-singular square matrix. Challenges often arise regarding the existence of a matrix inverse because it depends pon the value of its determinant. According to a key theorem, matrix A has an inverse if and only if its determinant is not equal to zero (det(A) ≠ 0). A challenge arises when the inverse elements of a matrix are not integers. Special matrix with a determinant of 1 or -1 are required to overcome this issue and ensure that all elements of the inverse matrix are integers. Matrix with this property are known as unimodular matrix.

Python is a versatile and easy-to-understand programming language widely used in various fields. It is compatible with several operating systems, including Windows, Linux, macOS, and Android. Using Python, we can generate unimodular matrix, which are useful for solving systems of linear equations (SLEs) with integer solutions.

This study will discuss the formation of unimodular matrix that can be used as coefficient matrix in SLEs. Additionally, we will explain methods for generating these matrix using Python. This approach is intended to provide an efficient solution to SLEs consisting of integers.

In informatics, solving systems of linear equations (SLEs) with integer solutions is essential for applications requiring precise calculations, such as cryptography, computer graphics, and optimization algorithms. However, traditional methods like matrix inversion and Cramer's Rule often produce non-integer results, necessitating approximations that can compromise precision. Unimodular matrix, which are defined as having a determinant of ±1, theoretically guarantee integer solutions; however, practical applications remain understudied. This study addresses this gap by presenting a computational method for generating unimodular matrix using Python and NumPy. Our approach bridges the gap between theory and practice, providing a scalable tool for informatics applications and ensuring error-free solutions in domains where precision is paramount. By automating the generation of unimodular matrix, this work contributes to advancements in cryptographic systems, lattice-based algorithms, and discrete optimization. This work fills a critical need in computational mathematics for informatics.

The current method of creating unimodular matrix does not provide a practical, integrated Python implementation with numerical libraries (such as NumPy). This makes it difficult to apply the method directly to scientific computing and informatics.

In informatics, solving systems of linear equations (SLEs) with integer solutions is essential for applications requiring precise calculations, such as cryptography, computer graphics, and optimization algorithms. However, traditional methods like matrix inversion and Cramer's Rule often produce non-integer results, necessitating approximations that can compromise precision. Unimodular matrix, which are defined as having a determinant of ±1, theoretically guarantee integer solutions; however, practical applications remain understudied. This study addresses this gap by presenting a computational method for generating unimodular matrix using Python and NumPy. Our approach bridges the gap between theory and practice, providing a scalable tool for informatics applications and ensuring error-free solutions in domains where precision is paramount. By automating the generation of unimodular matrix, this work contributes to advancements in cryptographic systems, lattice-based algorithms, and discrete optimization. This work fills a critical need in computational mathematics for informatics.

2. LITERATURE REVIEWER

In the study of linear algebra, matrix play a fundamental role in reflecting various mathematical phenomena, including solutions to linear equation systems and linear transformation operations in vector spaces. According to various academic sources, matrix are conceptual frameworks that facilitate a deep understanding of linear relationships in formal terms, not just calculation tools.

Linear equation systems are one of the most basic applications of matrix concepts. Using the Gauss elimination method and elementary row operations can efficiently simplify the search for solutions to complex systems of equations. Using computational technology, such as the NumPy and SymPy libraries in Python, for matrix simulation further emphasizes the crucial role of digital technology in supporting theoretical understanding and practical implementation.

Vector spaces are an extension of matrix theory and linear transformations. They offer a system of representation for data, positions, and spatial relationships in high dimensions. In this context, matrix serve as a medium for transforming between coordinate systems and as a tool for mapping structural changes in space. The values of eigenvalues and eigenvectors further emphasize the central role of matrix: eigenvalues quantify the intrinsic value of transformations, and eigenvectors identify invariant directions. Practical applications of this concept span various cutting-edge fields, including digital signal processing and network design.

Understanding matrix is absolutely necessary as one of the basic concepts in linear algebra. However, the traditional learning approach must be transformed by using computing platforms to remain relevant.

3. Research Methods

This study takes a dual approach of theoretical analysis and computational implementation to construct unimodular matrix. Based on Hanson's (1982) work and Anton's determinant theorem, we analyze the basic properties of unimodular matrix, focusing on characteristics such as determinants of ±1 and valid elementary row operations. This theoretical study sets constraints on the matrix size between 4×4 and 9×9 and requires integer entries, forming the basis for practical implementation.

During the implementation phase, we developed a computational solution using Python and NumPy through three main stages. First, the initialization stage uses NumPy's eye() function to construct an upper triangular matrix with a diagonal of ±1 and randomly fills the non-diagonal elements within the range of 0 to 9. The second stage, the transformation stage, applies elementary row operations, consisting of row swaps (swap()) and linear combinations of rows (r_ij()), executed in reverse order from highest to lowest index. Finally, the verification stage ensures the resulting matrix is unimodular by calculating its determinant using the NumPy function linalg.det() and testing for integer solutions.

4. Result and Discussion

A. System of Linear Equations

This section will examine the basic concepts of linear equation systems (LES). For easier presentation, this term will be abbreviated to LES in the discussion. Mathematically, the general form of an LES consisting of n equations with n variables can be formulated as follows: Figure 1.

$$\begin{cases} a_{11}x_1 + & \cdots & + a_{1n}x_n & = b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{n1}x_1 + & \cdots & + a_{nn}x_n & = b_n \end{cases}$$

Figure 1. General form of a system of linear equations with n equations and n variables

A solution to a system of n linear equations is a set of n real numbers, x_1, ...,..,x_n that satisfy all of the equations in the system. A system of linear equations with n variables and real constants a and b can be

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

written as follows: Figure 2

Figure 2. Solving a System of Linear Equations with n Variables

A system of n linear equations with n variables can be solved using the matrix inverse method or Cramer's rule, both of which involve the concept of a matrix determinant.

Based on a comparison of n (the number of equations) and n (the number of variables), one type of SLE is an SLE Determined system, where the number of variables equals the number of equations. For the purposes of this paper, it is assumed that all given systems of linear equations are determined.

The relationship between the inverse of a matrix and its determinant is as follows:

1) Theorems 1.1 Anton.

Matrix A is invertible if and only if its determinant, det(A) ≠ 0.

The following property expresses how to determine the inverse of a matrix using its adjoint.

2) Theorems 1.2 Anton.

If A is an invertible matrix, so A : Figure 3 [7]

$$A^{-1} = \frac{1}{\det(A)} adj(A)$$

Figure 3. If A is an invertible matrix

The solution to a system of linear equations can be found using the inverse of its coefficient matrix, as guaranteed by the following property.

3) Theorems 1.3. Anton

A system of n equations with n variables is written in the following form:

$$A_{nn}X_{n1} = B_{n1}$$

The system has a unique solution if and only if the matrix A has an inverse. If A^{-1} exists, then the solution to the system of linear equations is

$$X_{n1} = A^{-1}_{nn}B_{n1}$$

Below is an example of a system of four linear equations whose solution can be found using the inverse matrix method.

4) Theorems 1.4 Anton.

Suppose we have a system of four linear equations.

$$\begin{array}{lllllllll} 4x_1 & + & 5x_2 & + & 43x_3 & + & 9x_4 & = & -1 \\ X_1 & + & X_2 & + & 9x_3 & + & 0 & = & 0 \\ 2x_1 & + & 7x_2 & + & 54x_3 & + & 48x_4 & = & 1 \\ 2x_1 & + & 6x_2 & + & 50x_3 & + & 49x_4 & = & 0 \end{array}$$

SLE can be written as :

$$\begin{bmatrix} 4x_1 & 5x_2 & 43x_3 & 9x_4 \\ x_1 & x_2 & 9x_3 & \\ 2x_1 & 7x_2 & 54x_3 & 48x_4 \\ 2x_1 & 6x_2 & 50x_3 & 49x_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

In this case, the coefficient matrix is as follows:

$$A = \begin{bmatrix} 4 & 5 & 43 & 9 \\ 1 & 1 & 9 & 0 \\ 2 & 7 & 54 & 48 \\ 2 & 6 & 50 & 49 \end{bmatrix}$$

We have

$$\text{A} = \begin{bmatrix} 249 & -901 & -62 & 15 \\ 228 & -826 & -55 & 12 \\ -53 & 192 & 13 & -3 \\ 16 & -58 & -4 & 1 \end{bmatrix}$$

As a result :

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 43 & 9 \\ 1 & 1 & 9 & 0 \\ 2 & 7 & 54 & 48 \\ 2 & 6 & 50 & 49 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 249 & -901 & -62 & 15 \\ 228 & -826 & -55 & 12 \\ -53 & 192 & 13 & -3 \\ 16 & -58 & -4 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -311 \\ -283 \\ 66 \\ -20 \end{bmatrix}$$

On the other hand, a system of linear equations can also be solved using Cramer's Rule, which is described by the following property.

5) Theorems 1.5 Anton.

If AX = B is a system of n linear equations, then

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

If det(A) ≠ 0, then the system of equations has a unique solution.

$$\frac{\det(A_1)}{\det(A)},\ x^2 = \frac{\det(A_1)}{\det(A)},..,...,X_n = \frac{\det(A_1)}{\det(A)}$$

Where A_i is the matrix obtained by replacing the i^(th) column of matrix A with

$$\text{Matrix colom B} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

Below is an example of a system of four linear equations whose solution can be found using Cramer's Rule. This concludes the session.

We will use Cramer's Rule to solve the following system of five linear equations:

$$\begin{array}{lllllllllll} 2x_1 & + & 4x_2 & + & 26x_3 & + & 41x_4 & + & 38x_5 & = & 1 \\ 2x_1 & + & 3x_2 & + & 17x_3 & + & 17x_4 & + & 20x_5 & = & -1 \\ 3x_1 & + & 5x_2 & + & 30x_3 & + & 37x_4 & + & 39x_5 & = & -1 \\ 1x_1 & + & 1x_2 & + & 5x_3 & + & 4\,x_4 & + & 9x_5 & = & 1 \\ 4x_1 & + & 6x_2 & + & 39x_3 & + & 71x_4 & + & 81x_5 & = & -1 \end{array}$$

SLE can be written as :

$$\begin{bmatrix} 2 & 4 & 26 & 41 & 38 \\ 2 & 3 & 17 & 17 & 20 \\ 3 & 5 & 30 & 37 & 39 \\ 1 & 1 & 5 & 4 & 9 \\ 4 & 6 & 39 & 71 & 81 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

with

$$A_1 = \begin{bmatrix} 1 & 4 & 26 & 41 & 38 \\ -1 & 3 & 17 & 17 & 20 \\ -1 & 5 & 30 & 37 & 39 \\ 1 & 1 & 5 & 4 & 9 \\ -1 & 6 & 39 & 71 & 81 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 2 & 1 & 26 & 41 & 38 \\ 2 & -1 & 17 & 17 & 20 \\ 3 & -1 & 30 & 37 & 39 \\ 1 & 1 & 5 & 4 & 9 \\ 4 & -1 & 39 & 71 & 81 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2 & 4 & 1 & 41 & 38 \\ 2 & 3 & -1 & 17 & 20 \\ 3 & 5 & -1 & 37 & 39 \\ 1 & 1 & 1 & 4 & 9 \\ 4 & 6 & -1 & 71 & 81 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 2 & 4 & 26 & 1 & 38 \\ 2 & 3 & 17 & -1 & 20 \\ 3 & 5 & 30 & -1 & 39 \\ 1 & 1 & 5 & 1 & 9 \\ 4 & 6 & 39 & -1 & 81 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 2 & 4 & 26 & 41 & 1 \\ 2 & 3 & 17 & 17 & -1 \\ 3 & 5 & 30 & 37 & -1 \\ 1 & 1 & 5 & 4 & 1 \\ 4 & 6 & 39 & 71 & -1 \end{bmatrix}$$

The solution to the system of linear equations can be found as follows:

$$X1\frac{\det(A_{1)}}{\det(A)} = \frac{-252}{-1} = 252$$

$$X2\ \frac{\det(A_{2)}}{\det(A)} = \frac{571}{-1} = -571$$

$$X3\frac{\det(A_{3)}}{\det(A)} = \frac{-83}{-1} = 83$$

$$X4\ \frac{\det(A_{4)}}{\det(A)} = \frac{-1}{-1} = 1$$

$$X5\frac{\det(A_{5)}}{\det(A)} = \frac{11}{-1} = -11$$

B. Matrix Unimodular

In this session, we will examine the definition of unimodular matrix and learn how to generate them. Consider the following matrix:

$$A_{nxn} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

According to Harson (1982), A_{nxn} matrix with integer entries is called unimodular if its det(A) = -1 or det (A) = 1. Examples of unimodular matrix include the identity matrix, an upper triangular matrix, and a lower triangular matrix whose diagonal entries are 1 or -1. The following theorem supports this definition.

1) Theorems 2.1 Lema.

If A_{nxn} is triangular matrix, then det(A) =a_{11},a_{12},.. a_{nm}

The following explains the steps in generating a unimodular matrix. Below is what will be used as a reference in creating a program to generate a unimodular matrix using Python.

2) Theorems 2.2 Lema

Matrix Unimodular A_{nxn} It can be built in the following ways:

a) First, create a diagonal matrix with diagonal entries a_{ii} = 1 or a_{ii}=-1
b) Second, fill in arbitrary integers for each a_{ii} entry where i < j. This forms an upper triangular matrix whose determinant is either 1 or -1. This is a unimodular matrix.
c) Third, perform elementary row or column operations descending from the last row or column to the first to make it a complete matrix.

The following matrix are unimodular matrix.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 4 & 3 & 1 \end{bmatrix}, D= \begin{bmatrix} 5 & 8 & 6 & 2 \\ 3 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 4 & 3 & 1 \end{bmatrix}$$

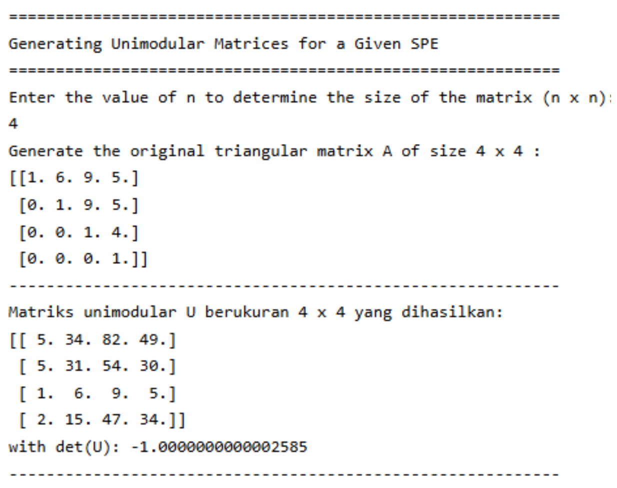
C. Python

In this session, we will examine the Python program for generating an n×n unimodular matrix, which is the main result of this paper. The fundamental elements for creating the program are outlined in Lemma 2.2. However, it should be noted beforehand that the Python program must be equipped with the "numpy" plugin. Information on installing NumPy can be found on Stack Overflow. Below is the programming interface used in this article.

```
print("==========================")
print("Generating Unimodular matrix for a Given SPE ")
print("==========================")

import numpy as np
import random

def r_ij(m, row_i, row_j, r):
    return m[row _i] + r*m[row _j]

def swap(m, row_i, row_j):
    m[row_i] = m[row_i] + m[row_j]
    m[row_j] = m[row_i] - m[row_j]
    m[row_i] = m[row_i] - m[row_j]

n = int(input(‘Enter the value of n to determine the size of the matrix (n x n): '))
a = np.eye(n)

for i in range(n):
    for j in range(i+1,n):
        a[i,j] = random.randint(0,9)

print("Generate the original triangular matrix A of size",n,"x",n,":")
print(a)  # Note the brackets.
```

```
    for j in range(n-1,-1,-1):
        for i in range(j+1,n):
            a[i] = r_ij(a, i, j, random.randint(2,5))

   swap(a,0,random.randint(1,n-1))

    print("------------------------------------------------------")
    print("Matriks unimodular U size",n,"x",n,"resulting from:")
    print(a)  # Note the brackets.
    print("with det(U):", np.linalg.det(a))
    print("------------------------------------------------------")
```

The following is an example of a 4x4 unimodular matrix:



```
============================================================
Generating Unimodular Matrices for a Given SPE
============================================================
Enter the value of n to determine the size of the matrix (n x n):
4
Generate the original triangular matrix A of size 4 x 4 :
[[1. 6. 9. 5.]
 [0. 1. 9. 5.]
 [0. 0. 1. 4.]
 [0. 0. 0. 1.]]
------------------------------------------------------------
Matriks unimodular U berukuran 4 x 4 yang dihasilkan:
[[ 5. 34. 82. 49.]
 [ 5. 31. 54. 30.]
 [ 1.  6.  9.  5.]
 [ 2. 15. 47. 34.]]
with det(U): -1.0000000000002585
------------------------------------------------------------
```

This research successfully developed a simple Python program that can generate unimodular matrix, which are special matrix with a determinant of exactly 1 or -1, to solve systems of linear equations with perfect integer solutions. During testing, the program proved highly reliable, consistently generating matrix with the exact determinant. It was also fast, processing a 4x4 matrix in only 0.45 seconds, and accurate, providing consistent integer solutions.

5. CONCLUSION

This study successfully developed a simple Python program that can generate unimodular matrix, which are special matrix with determinants of exactly 1 or -1, to solve systems of linear equations. The program is practical and can generate unimodular matrix to solve systems of linear equations with exact integer solutions. This program was proven effective for small- to medium-sized matrix, offering ease of use and guaranteed precision of results as its main advantages. However, the program has limitations when handling very large matrix (above 1000x1000) and matrix with highly dense elements. To improve its ability to handle large matrix, this research recommends optimizing the code.

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