

Matrix-Based Computation in Informatics: A Conceptual Review of Linear Algebra Applications

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ABSTRACT – Linear algebra and matrices play a critical role in informatics, especially in understanding multidimensional data and computational processes. This study aims to explore the fundamental concepts and practical applications of linear algebra and matrices using a descriptive qualitative method with a literature review approach. Data sources include textbooks, international journals, and recent academic articles. The findings reveal that concepts such as vector spaces, determinants, eigenvalues, and matrix transformations are widely applied in machine learning, image processing, and modern data analysis. Techniques like Principal Component Analysis (PCA) and Singular Value Decomposition (SVD) are proven effective for dimensionality reduction and feature extraction. The study concludes that linear algebra acts as a bridge between mathematical theory and informatics applications, serving as a foundation for developing intelligent systems and algorithms. This research is expected to serve as a conceptual reference for students and scholars in the field of information technology.

Keywords – Linear Equation, Linear Space, N-Tuples, Matrix and Linear Algebra

Komputasi Berbasis Matriks dalam Informatika: Tinjauan Konseptual atas Penerapan Aljabar Linier

ABSTRAK – Aljabar linier dan matriks memiliki peran penting dalam informatika, khususnya untuk memahami data multidimensi dan pemrosesan komputasi. Penelitian ini bertujuan mengkaji konsep dasar serta penerapan praktis aljabar linier dan matriks melalui metode kualitatif deskriptif dengan pendekatan studi pustaka. Sumber data diambil dari buku, jurnal internasional, dan artikel ilmiah terbaru. Hasil penelitian menunjukkan bahwa konsep seperti ruang vektor, determinan, nilai eigen, dan transformasi matriks digunakan secara luas dalam machine learning, pengolahan citra, serta analisis data modern. Teknik seperti Principal Component Analysis (PCA) dan Singular Value Decomposition (SVD) terbukti efektif untuk reduksi dimensi dan ekstraksi fitur. Studi ini menyimpulkan bahwa aljabar linier merupakan jembatan antara teori matematika dan penerapan informatika, serta menjadi dasar dalam pengembangan sistem cerdas dan algoritma. Penelitian ini diharapkan menjadi referensi konseptual bagi mahasiswa dan akademisi di bidang teknologi informasi.

Kata Kunci – Persamaan Linear, Ruang Lingkup Linear, N-Tuple, Matriks dan Aljabar Linear

1. INTRODUCTION

The foundations of linear algebra were established through the study of vectors in two- and three-dimensional Cartesian spaces. In this context, a vector is defined as a directed line segment characterized by its length and direction. Vectors can represent physical objects, and when combined with other vectors or multiplied by scalars, they form what is known as a vector space.

Recent advancements in linear algebra have expanded the concept to include spaces with any number of dimensions, even extending to infinity. The term "n-space" refers to a vector space with n dimensions, allowing for the generalization of many important results from two- and three-dimensional spaces to these higher-dimensional contexts. Although n -tuples or vectors can be challenging for humans to visualize in n -space, they are essential for data representation.

The element of a vector space is obvious, implying that any two bases of a similar space have a similar cardinality. The sole condition for a matrix to be invertible is for its determinant to be nonzero. Source: Singh (2021) In the event that the straight guide that a framework addresses is an isomorphism, the matrix may be inverted. See invertible matrix for additional comparable expressions; in the event that a square lattice has an opposite on the left or the right, it is invertible.

When all of a matrix's eigenvalues are positive or equal to zero, we say that the matrix is positive semidefinite. Each eigenvalue of a positive definite matrix must be bigger than zero for the matrix to be considered positive definite. If an $n \times n$ matrix has n directly autonomous eigenvectors, then it tends to be diagonalized, meaning that it can be transformed into an invertible network P and a corner to corner lattice D to such an extent that $A = PDP^{-1}$. A symmetric matrix is the only one that may be orthogonally diagonalizable, according to the spectral theorem.

As a completely abstract concept, vector spaces (also known as linear spaces) are crucial in abstract algebra. Notable examples include the ring of linear mappings in vector spaces and the group of invertible linear maps or matrices. Additionally, linear algebra plays an important role in the study of alternating maps, tensor products, and the explanation of higher-order derivatives in vector analysis.

In recent years, the role of linear algebra has become increasingly vital in cutting-edge research areas in informatics, such as artificial intelligence (AI), data mining, and computer vision. Matrix operations and vector transformations are widely used in neural networks, feature extraction, dimensionality reduction, and image processing techniques. These trends highlight the relevance of linear algebra not only as a mathematical discipline but also as a foundational tool in solving complex computational problems in modern data-driven technologies.

2. LITERATURE REVIEWER

The study of vectors in two-dimensional and three-dimensional Cartesian spaces marks the initial exploration of direct polynomial mathematics (Rathee, S. 2023). In this context, a vector is defined as a segment of a line characterized by its direction and magnitude, which are essentially two aspects of the same concept. A vector that has no positive or negative magnitude or direction is referred to as a zero vector. This represents the earliest known example of a real vector space, where the "scalars" are real numbers and the "vectors" represent physical phenomena such as forces. These vectors can be multiplied by scalars and combined to form the space (Spurio, 2023).

Many mathematicians consider vectors to be among the most abstract **concepts** (Aguirre and Erickson, 1984; Knight, 1995; Poynter and Tall, 2005). They are not only crucial for technological advancements in the 21st century but also foundational to linear algebra, encompassing areas like vector spaces, linear combinations, linear transformations, eigenvalues, and eigenvectors (Stewart et al., 2019). Modern linear algebra now includes spaces with arbitrary or infinite dimensions. A vector space with n dimensions is referred to as an n -space. Most practical results from two- and three-dimensional spaces can be extended to these higher-dimensional spaces.

Vectors or n -tuples serve as valuable representations of data, even though they can be difficult for humans to visualize in n -space. This framework is effective for summarizing and manipulating information, as vectors, represented as n -tuples, consist of n components. For example, in economics, one can create and use 8-dimensional vectors or 8-tuples to represent the Gross Domestic Product (GDP) of eight different countries. For a specific year, one might represent the GDP of the United States, the United Kingdom, Armenia, Germany, Brazil, India, Japan, and Bangladesh using a vector where each country's GDP occupies its designated position.

3. RESEARCH METHODS

This research employs a descriptive qualitative method with a literature review approach, utilizing various academic sources such as textbooks, international journals, and recent scientific articles relevant to linear algebra and matrices. The study constructs a conceptual framework that links the core concepts of linear algebra to their practical implementations in the field of informatics. For example, vector spaces and n -tuples are essential in representing high-dimensional data, which is a common requirement in data science and machine learning. Matrix operations such as multiplication, inversion, and transformation are fundamental in solving systems of linear equations, optimizing algorithms, and conducting structured data processing.

Furthermore, eigenvalues and eigenvectors serve as the foundation of dimensionality reduction techniques like Principal Component Analysis (PCA) and Singular Value Decomposition (SVD), which are widely used in pattern recognition, image compression, and feature extraction. Linear transformations are applied in computer vision for tasks such as image rotation, scaling, and noise reduction. These mathematical principles are implemented in various computational tools and frameworks like NumPy, TensorFlow, and PyTorch,

which are integral to the development of intelligent systems, neural networks, and data-driven applications. By synthesizing classical theories and modern technological practices, this study presents a comprehensive understanding that bridges theoretical mathematics with real-world informatika applications, demonstrating the essential role of linear algebra in advancing digital systems and computational innovation.

In addition to its computational significance, linear algebra also plays a strategic role in various informatics subfields such as cryptography, natural language processing (NLP), and network analysis. For instance, matrix operations are used in encryption algorithms and error detection methods, while vector representations such as word embeddings rely on linear transformations to capture semantic relationships in textual data. In network science, adjacency matrices are utilized to model and analyze complex systems like social media graphs and communication networks. The universality of linear algebra as a modeling and analytical tool enables it to serve as a bridge between mathematical theory and software engineering, making it an indispensable component in both academic research and industrial applications of informatics.

4. RESULT AND DISCUSSION

4.1 Linear Equation

Algebraic equations where each term is a constant or the product of a constant and a single variable are called linear equations. A linear equation may contain one or more variables. Linear equations are prevalent in the field of science and its various subfields, particularly in applied mathematics. The assumption that values of interest change only slightly from some "baseline" condition allows many nonlinear equations to be approximated as linear equations, which is very useful.

These equations naturally arise when modeling various phenomena. In linear equations, exponents are not considered. In this discussion, we take on the role of a researcher seeking the actual solutions to a single equation. This includes not only complex solutions but also covers linear equations with coefficients and arrangements in a broader sense across any field.

$$\begin{cases} 5x + y = 3 \\ 2x - y = 4 \end{cases}$$

It can be described in matrix form as follows Table 1.

Table 1. Matrix Representation of Linear Equations

Matrix A (Coefficient)		Vector X (Variables)	Vector B (Result)
5	1	x	3
2	-1	y	4

Or in matrix notation:

$$A \cdot x = b \quad \text{dimana} \quad \begin{bmatrix} 5 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

This system can be solved using the matrix inverse method or Gaussian elimination, which is an important part of data processing in numerical programming, machine learning, and algorithm optimization.

4.2 Matrix

The rectangular array of numerical values (or other mathematical objects) that constitute a matrix is referred to as its entries. According to Bernstein (2009), matrices can be easily manipulated using common mathematical operations such as addition and multiplication.

$$A = \begin{bmatrix} -1.3 & 0.6 \\ 20.4 & 5.5 \\ 9.7 & -6.2 \end{bmatrix}$$

$$\begin{matrix} & 1 & 2 & \dots & n \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \end{matrix}$$

As seen on the right, a matrix (plural: matrices, or less often just matrices) is a numerical array laid out in a rectangular fashion. Tables containing numerical data derived from physical observations are a common place to see matrices, but you may also find them in other mathematical settings.

For instance, as we'll learn in the next chapter, all the necessary data to resolve a system of equations can be organized into a matrix. This allows for efficient computation and manipulation of the equations involved. Matrices are fundamental in various fields, including physics, engineering, computer science, and economics, as they provide a structured way to represent and analyze data.

$$\begin{aligned} 5x + y &= 3 \\ 2x - y &= 4 \end{aligned}$$

Embedded in the matrix, and that the system's solution may be derived by applying the right

operations to this matrix.

$$\begin{bmatrix} 5 & 1 & 3 \\ 2 & -1 & 4 \end{bmatrix}$$

Tensors are arrays of numbers with more dimensions than one dimension, such as three dimensions, whereas vectors are matrices with only one column or row. It is possible to multiply matrices according to a method that compares to the creation of straight changes, and to add and remove them entry by entry. Except for the fact that matrix multiplication is not commutative, these operations meet the standard identities; nonetheless, the identity $AB=BA$ might fail.

The representation of linear transformations in matrices is one use case. These transformations are higher-dimensional versions of the same linear functions, where c is a constant and $f(x) = cx$. Another useful use of matrices is to record the values of the coefficients in a set of direct conditions. Settling an arrangement of straight conditions with a square lattice is constrained by the determinant and, if present, the opposite network. The geometry of the related linear transformation may be understood from the eigenvalues and eigenvectors. There are a lot of uses for matrices. Several areas of physics make use of them, including matrix mechanics and geometrical optics.

4.2.1 Addition And Subtraction Of Linear Matrices

Matrix A and B's sum (difference) is the matrix that results from adding (subtracting) the elements at corresponding positions of A and B. Since the rules for adding and subtracting linear matrices only apply to matrices of equal order, Thus,

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 3 & -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 3 & -3 \end{bmatrix}$$

$$\Rightarrow A+B = \begin{bmatrix} 0 & 6 & 5 \\ 7 & 2 & -3 \end{bmatrix} \text{ and } A-B = \begin{bmatrix} 2 & 2 & -1 \\ -1 & -4 & 3 \end{bmatrix}.$$

However, if

$$C = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix},$$

Consequently, C is an element that cannot be combined with A or B in any way. As a fundamental concept in linear algebra, matrices are closely related to linear transformations. In addition to rings and elements from more broad mathematical domains, there are other sorts of entries that are used.

4.2.2 Transformation To Matrix

As an example, a 2×1 column vector may be obtained by pre multiplying it with a 2×2 matrix:

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 17 \\ -9 \end{bmatrix}$$

If the vector $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$ The matrix has transformed the point with coordinates (7, -1) into the new point with coordinates (17, -1), which is represented by the position vector I. In a similar vein, the matrix influences every single point on the plane. This may be expressed as T, the transformation,

$$T = \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Image points (x', y') are transformed into point (x, y) using T.

Based on the matrix provided

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x + 4y \\ -x + 2y \end{bmatrix}$$

Additionally, the transformation may be expressed in the following way: T: $x' = 3x + 4y$, $y' = -x + 2y$.

4.2.3 Matrix Multiplication

The conditions for the definition of matrix multiplication are that the left matrix must have an equal number of segments and the right lattice should have an equivalent number of lines. When A and B are m -by- n and n -by- p lattices, individually, the m -by- p matrix AB is obtained by multiplying them.

The elements of this matrix are supplied by: where $1 \leq i \leq m$ and $1 \leq j \leq p$. [5] We must consider the vector as a column matrix in order to specify the matrix-vector product, which is the multiplication of a matrix A with a vector x . The network vector item is characterized exclusively for the circumstance when the quantity of sections in A is equivalent to the quantity of lines in x .

Thus, for $n \times 1$ column vectors x , the product Ax is defined if A is a $m \times n$ matrix (i.e., with n columns). The vector b is a $m \times 1$ column vector if we let $Ax = b$. Put otherwise, the product of b and A is characterized by the quantity of lines in A, which might be anything.

One common way to calculate the matrix-vector product is

$$Ax = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{bmatrix}$$

The procedure of matrix-vector multiplication is really very straightforward, despite its seeming complexity. For each row of A, one obtains the dot product of x . For this reason, it is essential that the quantity of sections in A be equivalent to the quantity

of parts in x . In the network vector item, the initial segment is the spot result of x and the main line of s , and so on. really, the matrixvector product is really just a dot product masquerading as A if A has a single row.

4.2.4 Linear Equations

Babylonians, some 4,000 years ago, could solve a basic system of two-by-two linear equations involving just two variables. An ability to tackle a 3x3 arrangement of conditions was shown by the Chinese in 200 BC when they produced "Nine Chapters of the Mathematical Art" (Perotti). As far back as recorded history goes, individuals from all walks of life have attempted to solve the seemingly easy equation $ax+b=0$. Late in the 17th century was when linear algebra's strength and advancements were realized.

Turn of the nineteenth century saw the introduction of a technique for tackling frameworks of straight conditions via Carl Friedrich Gauss [Ferreirós, J. (2020)]. While he did touch on linear equations in his writings, matrices and their notations were still outside of his purview. Different equations involving different numbers and variables were the focus of his study, along with the classics from the pre-modern era by Euler, Leibnitz, and Cramer. To summarize Gauss's work, the phrase "Gaussian elimination" is now used. Some equations may have their variables removed using this technique, which relies on the ideas of merging, exchanging, or multiplying rows with each other. Once the variables have been discovered, the remaining unknown variables may be found by using back substitution [Ali et. al., 2021].

An illustration of grid increase that is firmly connected with straight conditions is the point at which A_n is a m -by- n framework and x is a segment vector (i.e., a $n \times 1$ -lattice) with n factors x_1, x_2, \dots, x_n . For this situation, the grid condition Hatchet = b , where b is a $m \times 1$ - section vector, is equivalent to the arrangement of straight conditions (Bronson, Richard, 1989).

$$\begin{aligned} A_{1,1}x_1 + A_{1,2}x_2 + \dots + A_{1,n}x_n &= b_1 \\ A_{m,1}x_1 + A_{m,2}x_2 + \dots + A_{m,n}x_n &= b_m \end{aligned}$$

Matrix algebra allows for the concise representation and handling of frameworks of straight conditions, which comprise of a few direct conditions.

5. CONCLUSION

Contemporary physics relies heavily on linear transformations and the corresponding symmetries. The use of quantum theory to the study of chemical bonding and spectroscopy has greatly expanded the range of applications for matrices in chemistry. We provide here a mathematical study of matrices and

linear algebra. When an algebraic equation has terms that are either constants or the product of a constant and one variable raised to the power of one, we say that the equation is linear. It is possible for linear equations to have many variables. Linear algebra delves into a set of linear equations, vector spaces (sometimes called linear spaces), linear mappings (also called linear transformations), and vectors.

In the context of information technology, the concepts of linear algebra and matrices are recommended to be applied more broadly in areas such as machine learning, data compression, cryptography, and image processing. Techniques such as Principal Component Analysis (PCA), Singular Value Decomposition (SVD), and matrix factorization are essential in building intelligent systems that require large-scale data analysis, pattern recognition, and dimensionality reduction. Therefore, it is highly encouraged that the implementation of these mathematical foundations be integrated into the development of software tools, algorithms, and data-driven platforms across industries.

However, this study is limited by its reliance on secondary data through literature review and does not include empirical testing or algorithmic implementation. Future research could explore computational simulations, case studies, or the development of prototype systems that embed linear algebra operations into real-world applications. Additionally, deeper investigations into the performance comparison of different matrix-based methods in specific informatics scenarios (image classification, data clustering, or encryption) would enrich both the theoretical and practical contributions of this field.

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