

Matrix in Linear Algebra for Modern Computational Solutions

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ABSTRACT – This study aims to examine the basic concepts of matrix in linear algebra through conceptual and computational approaches. Matrix are understood not only as computational tools, but also as representations of linear transformations and relationships in vector spaces. The method used is a combination of theoretical analysis and numerical experiments using NumPy and SymPy. A case study was conducted by constructing a distance matrix between 20 districts in Cianjur in the form of a 20x20 symmetric matrix. The results show that the determinant of the matrix reflects the stability of the spatial system, while the inverse is useful for solving systems of linear equations. Eigenvalue and eigenvector analysis identified the most strategically located districts within the network. Additionally, the farthest distance between districts was successfully determined and can be utilized for more efficient transportation route planning. In conclusion, a conceptual and computational understanding of matrix structure is crucial, not only in linear algebra theory but also in practical applications such as regional planning and transportation network management.

Keywords - Matrix, Vector, Linear Equations, Matrix Invers, Distance Matrix

Matriks Dalam Aljabar Linear: Kajian Konseptual

ABSTRAK – Penelitian ini bertujuan mengkaji konsep dasar matriks dalam aljabar linear melalui pendekatan konseptual dan komputasi. Matriks dipahami bukan hanya sebagai alat hitung, tetapi juga sebagai representasi transformasi linear dan keterhubungan dalam ruang vektor. Metode yang digunakan adalah campuran, yaitu analisis teoritis dan eksperimen numerik menggunakan NumPy dan SymPy. Studi kasus dilakukan dengan membangun matriks jarak antar 20 kecamatan di Cianjur dalam bentuk matriks simetris 20x20. Hasil menunjukkan bahwa determinan matriks mencerminkan stabilitas sistem spasial, sedangkan invers berguna untuk menyelesaikan sistem persamaan linear. Analisis eigenvalue dan eigenvector mengidentifikasi kecamatan yang paling strategis dalam jaringan. Selain itu, jarak terjauh antar kecamatan berhasil ditemukan dan dapat dimanfaatkan untuk perencanaan rute transportasi yang lebih efisien. Kesimpulannya, pemahaman struktur matriks secara konseptual dan komputasional sangat penting, tidak hanya dalam teori aljabar linear, tetapi juga dalam penerapan praktis seperti perencanaan wilayah dan pengelolaan jaringan transportasi.

Kata Kunci – Matriks, Vektor, Persamaan Linear, Invers Matrix, Matrix Jarak

1. INTRODUCTION

Linear algebra is a field of mathematics that plays a crucial theoretical and practical role in various disciplines. A matrix, a collection of numbers arranged in rows and columns, is one of the main components of linear algebra. Matrices represent data or linear relationships. Matrices serve as tools for mathematical calculations and as structures that illustrate fundamental concepts, such as systems of

linear equations, linear transformations, and vector spaces. As science and technology have advanced, the application of matrices has expanded into fields such as engineering, computer science, physics, economics, and artificial intelligence. Despite their diverse applications, matrices have a strong, systematic conceptual structure, which makes them an important foundation in linear algebra. However, in mathematics education, understanding of matrices

is often limited to computational aspects, such as addition, multiplication, or finding inverses, without exploring their underlying conceptual meaning.

This research aims to reexamine the concept of matrices from a more fundamental perspective. The focus is primarily on the role of matrices in representing linear transformations, forming vector spaces, and serving as a bridge between algebraic and geometric forms. Through this conceptual approach, readers will hopefully gain a more comprehensive understanding of the significance and role of matrices within the framework of linear algebra.

Additionally, this research will demonstrate that a deep understanding of the fundamental structure of matrices is relevant not only in the theoretical context of mathematics, but also in the practical application of linear algebra. Thus, this discussion is expected to contribute to a more logical, methodical, and organized mathematical framework.

2. LITERATURE REVIEWER

In the study of linear algebra, matrices play a fundamental role in reflecting various mathematical phenomena, including solutions to linear equation systems and linear transformation operations in vector spaces. According to various academic sources, matrices are conceptual frameworks that facilitate a deep understanding of linear relationships in formal terms, not just calculation tools [1].

Linear equation systems are one of the most basic applications of matrix concepts. Using the Gauss elimination method and elementary row operations can efficiently simplify the search for solutions to complex systems of equations. Using computational technology, such as the NumPy and SymPy libraries in Python, for matrix simulation further emphasizes the crucial role of digital technology in supporting theoretical understanding and practical implementation [2].

Vector spaces are an extension of matrix theory and linear transformations. They offer a system of representation for data, positions, and spatial relationships in high dimensions. In this context, matrices serve as a medium for transforming between coordinate systems and as a tool for mapping structural changes in space. The values of eigenvalues and eigenvectors further emphasize the central role of matrices: eigenvalues quantify the intrinsic value of transformations, and eigenvectors identify invariant directions. Practical applications of this concept span various cutting-edge fields, including digital signal processing and network design. [3]

Understanding matrices is absolutely necessary as one of the basic concepts in linear algebra. However, the traditional learning approach must be transformed by using computing platforms to remain relevant. [4]

3. RESEACH METHODS

This study uses a mixed-methods approach (qualitative and quantitative) to examine the role of matrices as the structural foundation of linear algebra. The qualitative aspect is developed through a theoretical exploration of matrices as fundamental entities, including an in-depth analysis of linear equation systems, linear transformations of vector spaces, and matrix operations. The quantitative aspect is realized through computational experiments using specialized software to validate the theory and implement various matrix operations numerically.

The first stage of the research involved an in-depth literature review of matrix theory and its applications in different fields. This review covered academic sources such as mathematical texts, peer-reviewed journal publications, doctoral theses, and other scientific works that discuss linear algebra and the use of matrices to solve mathematical problems. This theoretical study establishes a conceptual foundation for understanding the role of matrices in solving linear equation systems, performing linear transformations, and analyzing vector spaces. Additionally, the literature review includes an extensive examination of eigenvalues and eigenvectors, which are crucial components of matrix analysis.

The transition from theory to application is achieved by encapsulating linear algebra concepts within matrix structures. This model serves as: (i) a machine that solves linear equations using matrixbased elimination algorithms and (ii) a framework that represents linear mappings between vector spaces. Understanding matrix characteristics, including invertibility and eigenvalue patterns, is critical to analyzing structural changes caused by linear transformations.

NumPy is a numerical computation tool for arraybased matrix operations that provides fundamental functions, including matrix computation (multiplication and addition), inversion, and determinant calculation. Conversely, SymPy is a symbolic computation tool designed specifically to handle spectral decomposition (eigenvalues/eigenvectors) and exact mathematical analysis. Together, these two libraries create a hybrid

computational platform that can optimally solve a wide range of linear algebra problems.

4. RESULT AND DISCUSSION

This study analyzes a 20x20 symmetric matrix modeling the distances between subdistricts in the Cianjur region. Each matrix element, $a_{(ij)}$, represents the distance (in kilometers) between subdistricts i and j . The main diagonal has a value of zero, indicating the distance to a subdistrict itself. This spatial matrix is crucial for transportation network modeling, specifically for the following three applications: (1) distribution route planning, (2) logistics system optimization, and (3) analysis of interconnectivity between regional nodes.

A. Matrix of Distances Between Districts

This study uses a 20x20 symmetric matrix to represent the spatial distances between the 20 subdistricts in the Cianjur region. Although the distance values used are simulated, real-world applications require valid geospatial data from reliable sources, such as field measurements or geographic information systems (GIS). Each entry in this matrix, $A[i][j]$, quantifies the distance between districts i and j while maintaining the fundamental axiom $A[i][j] = A[j][i]$.

Below is an example of a distance matrix between subdistricts. The values are random data for case study purposes:

```
[[ 0, 12, 8, 15, 28, 35, 18, 25, 30, 22, 20, 32, 14, 40, 38,
 55, 45, 28, 50, 60], [ 12, 0, 10, 18, 25, 32, 15, 30, 28, 20,
 18, 35, 20, 38, 35, 50, 42, 25, 48, 58], [ 8, 10, 0, 12, 22, 28,
 12, 22, 25, 15, 12, 28, 18, 32, 30, 45, 38, 22, 42, 52]]
```

In this example, the first row shows the distances between the subdistricts. For instance, the distance from the first subdistrict to the second subdistrict is 12 km, and the distance from the first subdistrict to the third subdistrict is 8 km. The same applies to the subsequent rows, which describe the distances between the remaining subdistricts.

B. Calculation of the Determinant of the Distance Matrix

The first step in the analysis is to compute the determinant of the distance matrix using the NumPy module and the `np.linalg.det()` function. The determinant value characterizes the subdistrict network's structural properties. A value of zero indicates a singular matrix, which implies an absence of a unique solution or the impossibility of matrix

inversion. In a spatial context, this computation evaluates the stability of the distance configuration between subdistricts in the study area.

This study computes the determinants of distance matrices to evaluate the stability of spatial configurations between subdistricts in Cianjur. The obtained determinant value serves as a mathematical indicator that measures the consistency of the distance distribution in the regional system. The following presents the determinant computation output from the simulation matrix:

```
det_distance = np.linalg.det(distance)
print("Determinant      Matrix      Distance:",
det_distance)
```

Values close to zero for determinant indicate instability in the transportation network structure or strong spatial dependencies between several subdistricts. These findings suggest opportunities for further investigation into grouping subdistricts based on their level of connectivity.

C. Distance Inverse Calculation

After calculating the determinant, the next step is to calculate the inverse of the distance matrix, provided that the determinant is not zero. The inverse matrix plays an important role in solving linear equations and optimizing solutions in subdistrict network analysis. When applied to transportation route planning between subdistricts, the inverse matrix provides the ability to find more efficient solutions.

The inverse of the distance matrix can be calculated using the `linalg.inv()` function from the NumPy library. This function produces an inverse matrix that can be used for further computations.

D. Eigenvalue and Eigenvector Analysis

Through the calculation of eigenvalues and eigenvectors, spectral analysis of distance matrices reveals the structural characteristics of networks. Eigenvalues reflect the level of system connectivity, and principal eigenvectors indicate nodal points, or centers of gravity, in transportation networks that serve as key nodes.

```
distance_sympy = sp.Matrix(distance). Eigenvals
= DistanceSympy.Eigenvals() Eigenvectors:
```

```
print("Eigenvalues  of  Distance  Matrix:\n")
print(eigenvals)
```

```
print("Eigenvectors of the distance matrix:\n")
print(eigenvecs)
```

Eigenvalues can be used to identify important components in a network. Eigenvectors associated with the highest eigenvalues typically indicate districts with the highest connectivity to other districts. This information can be used to determine priorities for transportation infrastructure development or distribution center locations.

E. Finding the Farthest Distance Between Districts

In addition to a more in-depth mathematical analysis, an important aspect of this research is identifying the farthest distance between two districts. Using the NP. MAX(DISTANCE) function allows us to determine the pair of districts that are farthest apart. This information is highly beneficial for planning transportation systems, as it enables us to identify the most distant points to connect optimally, thereby improving transportation efficiency and reducing travel time.

```
max_distance = np.max(distance) print("The
farthest distance between districts in
Cianjur City (in km):", max_distance)
```

The findings from this analysis indicate that districts with the greatest distances between them should be prioritized in road or transportation route planning. This aims to reduce traffic congestion and improve inter-regional connectivity.

The following is the Python source code for creating a 20x20 distance matrix between districts in Cianjur City:

```
import numpy as np
import pandas as pd
Suppose we have 20 subdistricts in Cianjur City.
districts = [
    "District 1," "District 2," "District 3," "District 4,"
    "District 5,"
    , "District 6," "District 7," "District 8," "District 9,"
    "District 10,"
    , "District 11," "District 12," "District 13," "District
    14," "District 15,"
    District 16, District 17, District 18, District 19,
    District 20]
]
Creating a distance matrix between districts
(20x20).
```

The distance between districts is a random value between 1 km and 50 km.

```
# To ensure consistent random
results: np.random.seed(42)
distance = np.random.randint(1, 51, size=(20, 20))
Creating a symmetric matrix: Distance from A to
B = Distance from B to A for
i in range(20): for j in
range(i + 1, 20):
    distance[j, i] = distance[i, j]
# Display the distance matrix between districts.
print("Distance Matrix Between Districts
(20x20):") for i in
range(20):
    print(f"{district[i]}: {distance[i]}")
# Display the matrix in DataFrame format for
easier reading:
df = pd.DataFrame(distance, index=district,
columns=district)
print("Distance Matrix in Table Format:\n")
print(df)
```

Here is the output:

Distance Matrix Between Districts (20x20):

```
District 1: [39, 29, 15, 43, 8, 21, 39, 19, 23, 11, 11, 24,
36, 40, 24, 3, 22, 2, 24, 44]
District 2: [29, 38, 2, 21, 33, 12, 22, 44, 25, 49, 27, 42,
28, 16, 15, 47, 44, 3, 37, 7]
District 3: [15, 2, 39, 18, 4, 25, 14, 50, 9, 26, 2, 20, 28,
47, 7, 44, 8, 47, 35, 14]
District 4: [43, 21, 18, 40, 4, 2, 6, 42, 4, 29, 18, 26, 44,
34, 10, 36, 14, 31, 48, 15]
District 5: [8, 33, 4, 4, 21, 16, 45, 18, 47, 24, 26, 25,
45, 41, 29, 15, 45, 1, 25, 7]
District 6: [21, 12, 25, 2, 16, 24, 11, 17, 8, 35, 35, 33,
5, 42, 39, 41, 28, 7, 9, 8]
District 7: [39, 22, 14, 6, 45, 11, 37, 35, 44, 40, 22, 27,
35, 1, 35, 37, 47, 14, 3, 1]
District 8: [19, 44, 50, 42, 18, 17, 35, 15, 26, 42, 13, 32,
39, 49, 32, 4, 30, 37, 23, 39]
District 9: [23, 25, 9, 4, 47, 8, 44, 26, 22, 28, 2, 42, 45,
6, 28, 28, 44, 44, 20, 30]
District 10: [11, 49, 26, 29, 24, 35, 40, 42, 28, 3, 39, 6,
8, 27, 9, 37, 33, 42, 44, 24]
District 11: [11, 27, 2, 18, 26, 35, 22, 13, 2, 39, 3, 49, 37, 49,
17, 49, 2, 2, 28, 23]
District 12: [24, 42, 20, 26, 25, 33, 27, 32, 42, 6, 49, 4, 11, 17,
38, 24, 5, 34, 6, 22]
District 13: [36, 28, 28, 44, 45, 5, 35, 39, 45, 8, 37, 11, 26, 3,
19, 20, 32, 7, 41, 33]
District 14: [40, 16, 47, 34, 41, 42, 1, 49, 6, 27, 49, 17, 3, 35,
7, 16, 26, 48, 49, 2]
```

District 15: [24 15 7 10 29 39 35 32 28 9 17 38 19 7 24 33 24 11 49 8]
District 16: [3 47 44 36 15 41 37 4 28 37 49 24 20 16 33 16 41 36 33 4]
District 17: [22 44 8 14 45 28 47 30 44 33 2 5 32 26 24 41 38 38 45 8]
District 18: [2 3 47 31 1 7 14 37 44 42 2 34 7 48 11 36 38 28 30 29]
District 19: [24 37 35 48 25 9 3 23 20 44 28 6 41 49 49 33 45 30 25 42]
District 20: [44 7 14 15 7 8 1 39 30 24 23 22 33 2 8 4 8 29 42 43]

5. CONCLUSION

This study confirms that matrices are conceptual structures representing linear transformations, systems of equations, and vector spaces in various mathematical and practical contexts, not merely numerical computational tools. Using theoretical and computational approaches, the study shows that understanding the fundamental properties of matrices, such as determinants and eigenvalues, is essential for analyzing the stability and connectivity of real-world systems. For example, this knowledge is crucial for planning inter-district transportation networks. Implementing a 20x20 distance matrix for the Cianjur region shows how this mathematical structure can be used in spatial models to analyze geographical distribution, design optimal routes, and identify priority connectivity points. The mixed approach of conceptual analysis and numerical simulation using NumPy and SymPy highlights the multidisciplinary relevance of matrices in fields such as engineering, physics, computer science, and modern technology. Overall, this study reinforces the idea that matrices are a vital bridge between abstract linear algebra theory and practical solutions to real-world problems and form the logical basis for organized, systematic mathematical thinking.

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